Chapter – 6

Triangles

Similar Figures

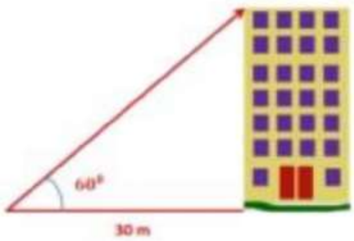
Introduction

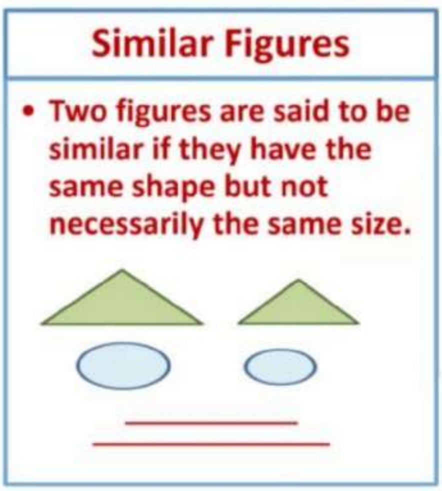
Applications of triangles in the real world are,

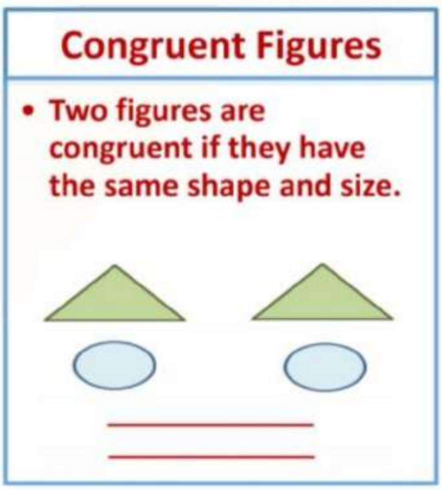
The triangular shape is the strongest shape and thus used extensively in architecture.

Triangles form the base unit of engineering structures like bridges.

Trigonometry, which is calculations with triangles can be used to measure the height of a building or a mountain.







If two figures are similar, then we can shrink or stretch the figure without changing its shape to obtain another similar shape of the figure.

Concept of similarity is used to measure the heights or distance of objects like mountain, planets.

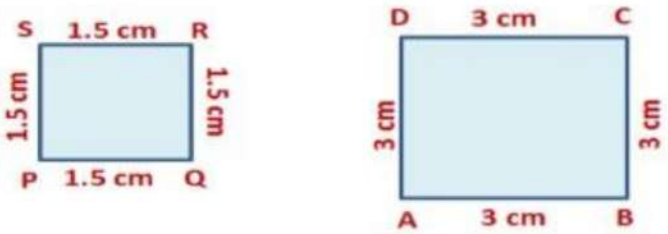
All congruent figures are similar but similar figures need not be congruent.

Two polygons of the same number of sides are similar if

- all the corresponding angles are equal.
- all the corresponding sides are in the same ratio

Ratio of corresponding sides is referred to as the scale factor (Representative Fraction) for the polygons.

Let's consider an example, here are two quadrilaterals PQRS and ABCD.



As the corresponding angles and ratio of corresponding sides are equal, the two quadrilaterals PQRS and ABCD are similar.

We write PQRS~ ABCD

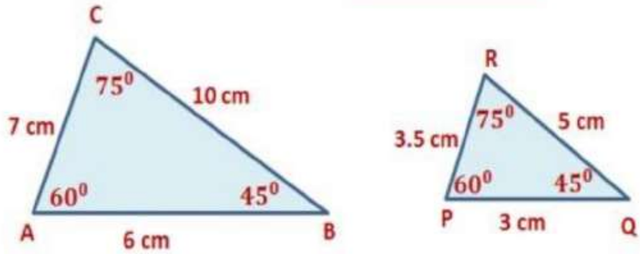
The symbol '~' stands for 'is similar to'

Similarity of triangles

Similarity of Triangles

Two triangles are said to be similar if their corresponding angles are equal and their corresponding sides are in the same proportion.

For example here are two triangles, Δ ABC and Δ PQR



In Δ ABC and Δ PQR

$$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$$

$\frac{AB}{PQ} = \frac{6}{3} = \frac{3}{1}, \frac{BC}{QR} = \frac{10}{5} = \frac{2}{1}, \frac{AC}{PR} = \frac{7}{3.5} = \frac{2}{1}$

Δ ABC and Δ PQR are similar as the ratio of their corresponding sides is same and corresponding angles are equal.

- If **corresponding angles of two triangles are equal**, then they are known as **equiangular triangles**.
- **The ratio of any two corresponding sides in equiangular triangle is always the same.**

Basic Proportionality Theorem or Thales Theorem

Theorem 1: If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Given: In ΔABC, DE∥BC and DE intersect AB at D and AC at E.

To Prove: $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Join BE and CD and draw EN⊥ AB and DM⊥AC

Proof: As EN⊥AB. EN is the height of △ADE and △DBE

We know, Area of a triangle = $\frac{1}{2}$ ×Base × Height

Now, area of Δ ADE = $\frac{1}{2}$ × AD × EN

Area of Δ BDE = $\frac{1}{2}$ ×DB×EN

Area of Δ ADE = $\frac{1}{2}$ ×AE×DM

Area of Δ DEC = $\frac{1}{2}$ ×EC×DM

The ratio of area of triangles, Δ ADE and Δ BDE

$$\frac{Area\ of\ \Delta ADE}{Area\ of\ \Delta BDE} = \frac{\frac{1}{2}\ X\ AD\ X\ EN}{\frac{1}{2}\ X\ EC\ X\ DM} = \frac{AD}{DB} \rightarrow \text{Eq 1}$$

$$\frac{Area\ of\ \Delta ADE}{Area\ of\ \Delta DEC} = \frac{\frac{1}{2}\ AE\ X\ DM}{\frac{1}{2}\ EC\ X\ DM} = \frac{AE}{EC} \rightarrow \text{Eq 2}$$

According to the property of triangles, the triangles drawn between the same parallel lines and on the same base have equal areas.

Therefore, the area of Δ BDE =area of Δ DEC →Eq 3

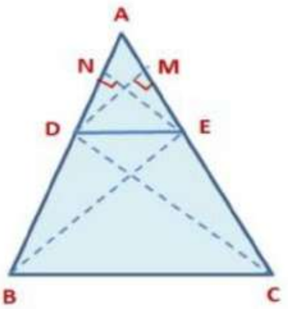
From equations 1, 2 and 3 we have, $\frac{AD}{DB} = \frac{AE}{EC}$

Theorem 2: If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

(Converse of Basic Proportionality)

Given: A ΔABC and a line DE, intersecting AB at D and AC at E such that, $\frac{AD}{DB} = \frac{AE}{EC} \longrightarrow$ Eq 1

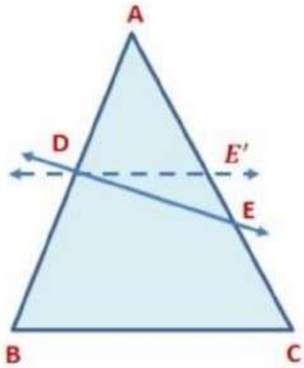
To Prove: DE ∥BC.



We will assume that in Δ ABC, DE is not parallel to BC. If DE is not parallel to BC, we draw DE’ parallel to BC.

Applying Basic Proportionality Theorem we get,

$$\frac{AD}{DB} = \frac{AE'}{E'C} \longrightarrow \text{Eq 2}$$



We will assume that in Δ ABC, DE is not parallel to BC. If DE is not parallel to BC, we draw DE’ parallel to BC.

Applying Basic Proportionality Theorem we get,

$$\frac{AD}{DB} = \frac{AE'}{E'C} \longrightarrow \text{Eq 2}$$

From Eq 1 and Eq 2, we get

$$\frac{AE}{EC} = \frac{AE'}{E'C} \longrightarrow \text{Eq 3}$$

Adding 1 to both sides of Eq 3, we get

$$\frac{AE}{EC}+1=\frac{AE'}{E'C}+1\Rightarrow\frac{AE+EC}{EC}=\frac{AE'+E'C}{E'C}$$

$$\frac{AC}{EC}=\frac{AC}{E'C}\Rightarrow EC=E'C$$

Therefore, we can say that the point E and F coincides. Hence, DE is also parallel to BC.

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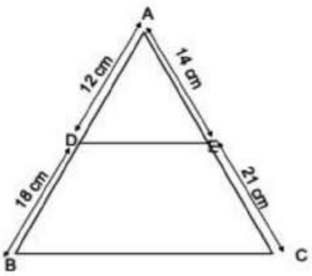
Example: If D and E are points on the respective sides AB and AC of Δ ABC such that AD = 12 cm, BD = 18 cm, AE = 14 cm, EC = 21 cm

Prove that DE ∥BC.



In Δ ABC,

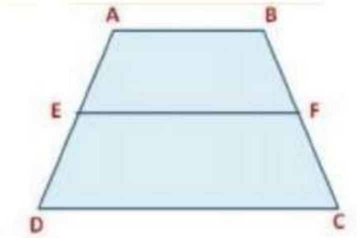
$\frac{AD}{DB}=\frac{12}{18}=\frac{2}{3}$ **and** $\frac{AE}{EC}=\frac{14}{21}=\frac{2}{3}$

Now, $\frac{AD}{DB}=\frac{AE}{EC}$

DE∥BC (By Converse of basic proportionality theorem)

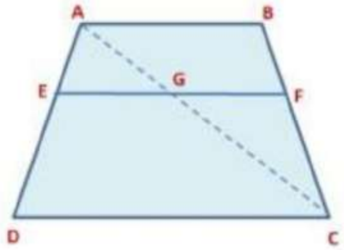
Example: ABCD is a trapezium with AB∥DC. E and F are two points on non-parallel sides AD and BC respectively, such that EF is parallel to AB. Show that $\frac{AE}{ED}=\frac{BF}{FC}$.

(REFERENCE: NCERT)



In trapezium ABCD, AB∥DC and EF ∥AB.

Now, join AC to intersect EF at G.



Here, EF ∥AB

(∵ lines parallel to the same line are parallel to each other)

In Δ ADC, EG∥DC (∵EF ∥DC)

So, $\frac{AE}{ED} = \frac{AG}{GC} \rightarrow$ Eq 1 (By Basic Proportionality Theorem)

In Δ ABC, GF∥AB (∵EF ∥AB)

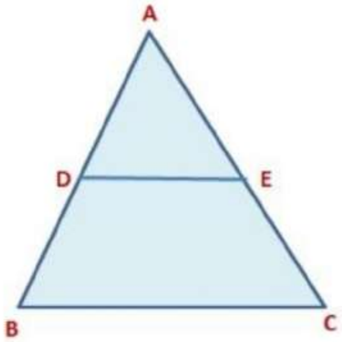
Similarly, $\frac{CG}{AG} = \frac{CF}{BF}$ (By Basic Proportionality Theorem)

On taking reciprocal of the terms we get, $\frac{AG}{GC} = \frac{BF}{CF} \rightarrow$ Eq 2

From Eq 1 and Eq 2 we get,

$$\frac{AE}{ED} = \frac{BF}{CF}$$

Example: In the given figure, DE ∥ BC. If AD = 3 cm. DB = 4 cm and AE =9 cm, find EC.



In Δ ABC, DE∥BC

So, $\frac{AD}{DB} = \frac{AE}{EC}$ (By Basic Proportionality Theorem)

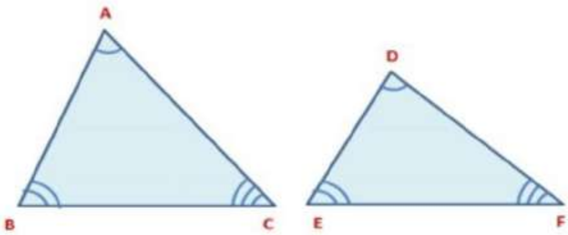
$$\frac{3}{4} = \frac{9}{EC} \Rightarrow EC = \frac{4 \times 9}{3} = 12$$

∴ EC = 12 cm

Criteria for similarity of triangles AAA Criterion

Criteria for Similarity of Triangles

∠A =∠D, ∠B = ∠E, and ∠C=∠F
$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$
We write similarity of these two triangles as Δ ABC~ ΔDEF



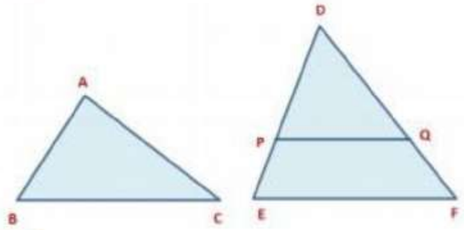
AAA Similarity Criterion

If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar.

We can prove this theorem by taking two triangles ABC and DEF.

Given: ∠ A =∠D, ∠ B =∠E and ∠ C = ∠F

To Prove: $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$ and then Δ ABC~ Δ DEF



Construction: Cut DP=AB and DQ= AC.

Join PQ.

Proof: In Δ ABC and Δ DEF

AB=DP	By Construction
AC=DQ	By Construction
$\angle$A =$\angle$D	Given
Δ ABC $\cong$ Δ DEF	By SAS Congruency Rule

$\therefore \angle B = \angle P$ (by CPCT)

But $\angle B = \angle E$ (given)

$\therefore \angle P = \angle E \Rightarrow PQ \| EF$ ($\because$ Corresponding angles are equal)

$\therefore \frac{DP}{PE} = \frac{DQ}{QF}$ (By Basic Proportionality Theorem)

On taking reciprocal we get,

$$\frac{PE}{DP} = \frac{QF}{DQ}$$

Adding 1 to both sides, we get

$$\frac{PE}{DP} + 1 = \frac{QF}{DQ} + 1 \Rightarrow \frac{PE + DQ}{DP} = \frac{QF + DQ}{DQ} \Rightarrow \frac{DE}{DP} = \frac{DF}{DQ}$$

$\Rightarrow \frac{DE}{AB} = \frac{DF}{AC}$ ($\because$DP=AB and DQ=AC)

On taking reciprocal we get,

$$\frac{AB}{DE} = \frac{AC}{DF}$$

$$Simelarly, \frac{AB}{DE} = \frac{BC}{EF} and, so \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$$

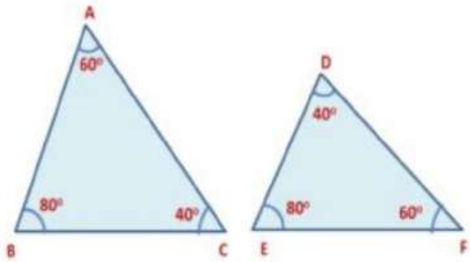
Hence, Δ ABC~ Δ DEF

If two angles of a triangle are respectively equal to two angles of another triangle, then by the angle sum property of a triangle their third angle will also be equal. Therefore AAA similarity criterion can be considered as AA similarity criterion.

Example: State whether the given pair of triangles are similar or not. In the case of similarity, mention the criterion.

In Δ ABC and Δ FED,

∠A =∠F = 60°
∠B =∠E = 80°
∠C =∠D = 40°
∴ Δ ABC ~ Δ FED by AAA similarity criterion

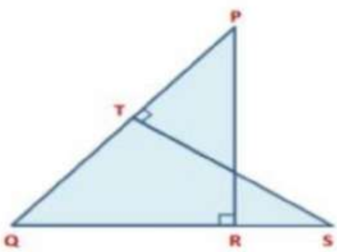


Example: In the given figure, PQR and QST are two right-angled triangles, right-angled at R and T, respectively. Prove that QR × QS= QP × QT

In Δ PRQ andΔSTQ

∠ PRQ =∠QTS	90°
∠ PQR =∠SQT	Common angle
Δ PQR~ Δ QST	By AA Similarity Criterion

Therefore, $\frac{QR}{QT} = \frac{QP}{QS}$



(Corresponding sides of similar triangles are proportional)

⇒ QR × QS= QP × QT Hence proved

Example: A girl of height 90 cm is walking away from the base of a lamp post at a speed of 1.2 m/s. If the lamp is 3.6 m above the ground, then find the length of her shadow after 4s.

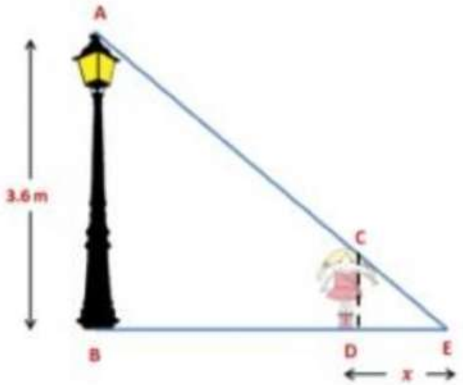
(REFERENCE: NCERT)

Let AB be the lamp-post, CD be the height of the girl and D be the position of the girl after 4 s.

Distance travelled by girl in 4 s = 1.2× 4 = 4.8 m

Here, AB = 3.6 m, CD = 90 cm = 0.9 m, BD = 4.8

In Δ ABE and Δ CDE



∠ B=∠D	90o
∠ E=∠E	Common angle
Δ ABE ~ Δ CDE	By AA Similarity Criterion

Therefore, $\frac{BE}{DE} = \frac{AB}{CD}$ (Corresponding sides of similar triangles are proportional)

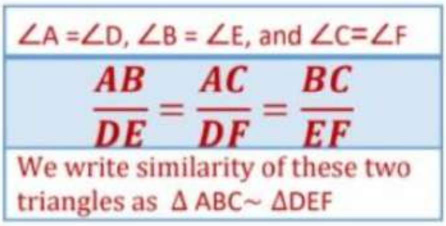
$$\frac{4.8 + x}{x} = \frac{3.6}{0.9} \Rightarrow \frac{4.8 + x}{x} = 4 \Rightarrow 4x = 4.8 + x \Rightarrow 3x = 4.8$$

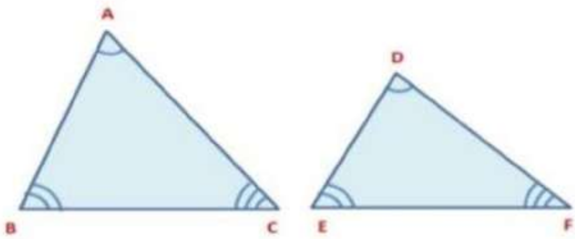
x = 1. 6

Hence, the length of her shadow is 1.6 m after 4 s.

Criteria for similarity of triangles - SSS Criterion

Criteria for Similarity of Triangles

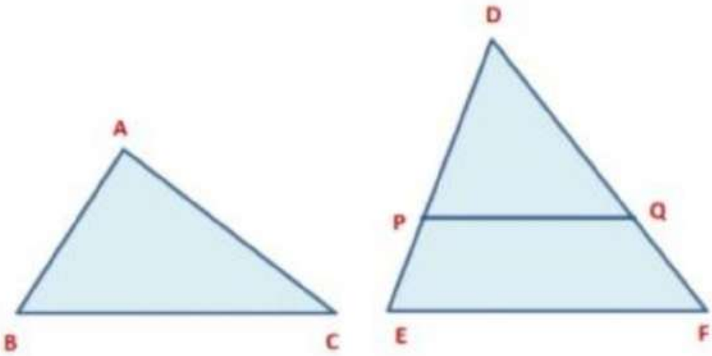




SSS Similarity Criterion

If in two triangles, sides of one triangle are proportional to the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.

We can prove this theorem by taking two triangles ABC and DEF.



Given: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} < 1 \rightarrow Eq1$

To Prove: $\angle A = \angle D, \angle B = \angle E and \angle C = \angle F$ and then $\triangle ABC \ \ \triangle DEF$

CONSTRUCTION : Cut DP=AB and DQ= AC. Join PQ.

Proof:

It is given that $\frac{AB}{DE} = \frac{AC}{DF} = \frac{DP}{DE} = \frac{DQ}{DF}$ (∵DP=AB and DQ=AC)

⇒ PQ∥EF (By Converse of Basic Proportionality Theorem)

$\therefore \angle DPQ = \angle E$ and $\angle DQP = \angle F \rightarrow$ Eq 2

($\because$ Corresponding angles are equal)

In Δ DPQ andΔ DEF

$\angle DPQ = \angle E$ and $\angle DQP = \angle F$

$\therefore$ ΔDPQ~ Δ DEF(By AA Similarity criterion)

$$\Rightarrow \frac{DP}{DE} = \frac{DQ}{DF} = \frac{PQ}{EF} \rightarrow \text{Eq3}$$

(Corresponding sides of similar triangles are proportional)

Putting AB = DP and AC = DQ in Eq 1 we get,

$$\frac{DP}{DE} = \frac{DQ}{DF} = \frac{BC}{EF} \rightarrow Eq4$$

In Δ ABC and Δ DPQ,

BC = PQ	From Eq 2 and Eq 3
AB = DP	By Construction
AC = DQ	By Construction
Δ ABC~ Δ DPQ	By SSS Congruency Rule

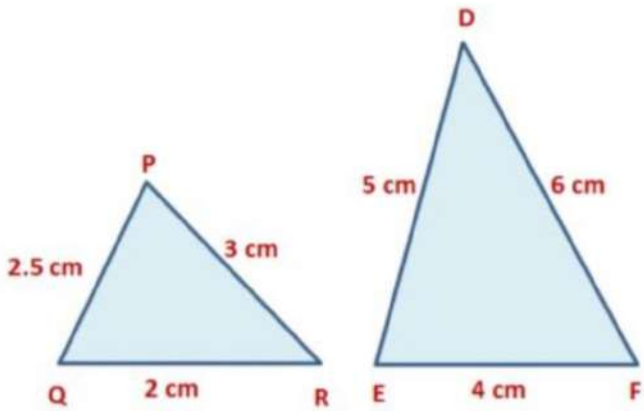
So, $\angle A = \angle D$, $\angle B = \angle DPQ$ and $\angle C = \angle DQP$ (By CPCT)

$\Rightarrow \angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$ (From Eq 2)

$\therefore$ Δ ABC~ Δ DEF (Hence Proved)

Example: For the given pair of triangles, state the similarity criterion used and write the relation in symbolic form.

In Δ PQR and Δ DEF,



In Δ PQR and Δ DEF,

$$\frac{PQ}{DE} = \frac{2.5}{5} = \frac{1}{2}$$

$$\frac{PR}{DF} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{QR}{EF} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{PQ}{DE} = \frac{PR}{DF} = \frac{QR}{EF} = \frac{1}{2}$$

∴ Δ PQR ~ Δ DEF by SSS Similarity Criterion

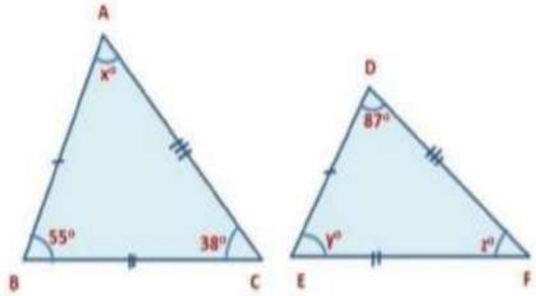
Example: Find the value of x^o, y^o, z^o in the given pair of triangles.

Given: AB = DE, BC = EF and AC = DF

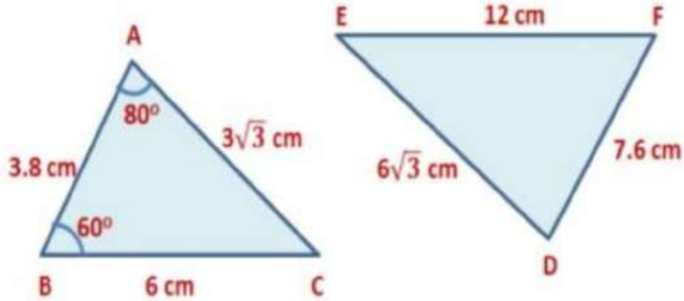
∴ Δ ABC ~ Δ DEF by SSS Similarity Criterion

So, ∠A = ∠D, ∠B = ∠E and ∠C = ∠F

$x^o = 87^o$, $y^o = 55^o$, $z^o = 38^o$



Example: In the given figure, find ∠F.



In Δ ABC and Δ DEF,

$$\frac{AB}{DF} = \frac{AC}{DE} = \frac{BC}{EF} = \frac{1}{2}$$

Δ ABC~ Δ DEF by SSS criterion of similarity

⇒ ∠A = ∠D, ∠B = ∠F and ∠C = ∠E

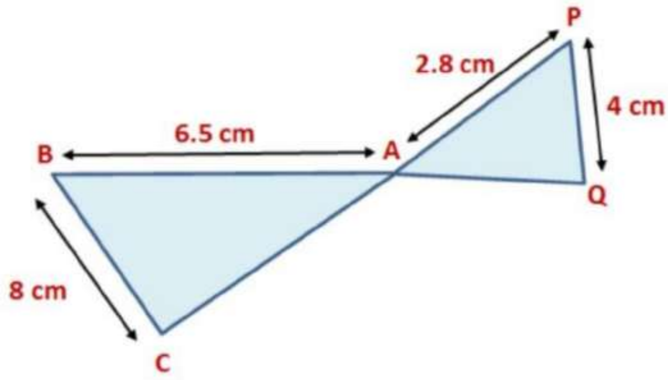
∴ ∠D = 80°, ∠F = 60°

Hence ∠F = 60°

Example: In the given figure, Δ ACB~ Δ APQ. If BC = 8 cm, PQ = 4 cm, BA = 6.5 cm, AP = 2.8 cm, find CA and AQ.

(REFERENCE: NCERT)

Here, Δ ACB ~ Δ APQ



$$\Rightarrow \frac{AC}{AP} = \frac{CB}{AQ} = \frac{AB}{AQ}$$

$$\Rightarrow \frac{AC}{AP} = \frac{CB}{PQ} \text{ and } \frac{CB}{PQ} = \frac{AB}{AQ}$$

$$\Rightarrow \frac{AC}{2.8} = \frac{8}{4} \Longrightarrow AC = \frac{2.8X8}{4} = 5.6$$

$$\Rightarrow \frac{8}{4} = \frac{6.5}{AQ} = AQ = \frac{4X6.5}{8} = 3.25$$

AC = 5.6 cm and AQ = 3.25 cm

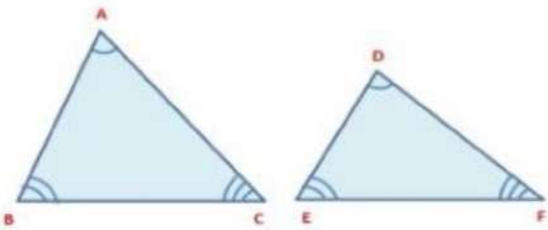
Criteria for similarity of triangles - SAS Criterion

Criteria for Similarity of Triangles

$\angle A = \angle D$, $\angle B = \angle E$, and $\angle C = \angle F$

$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$$

We write similarity of these two triangles as $\Delta ABC \sim \Delta DEF$



SAS Similarity Criterion

If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

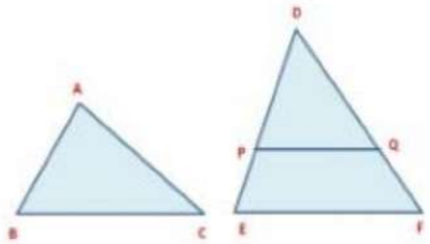
We can prove this theorem by taking two triangles ABC and DEF.

Given: $\frac{AB}{DE} = \frac{AC}{DF} < 1 \rightarrow$ Eq 1 and $\angle A = \angle D \rightarrow$ Eq 2

To Prove: Δ ABC~ Δ DEF

Construction: Cut DP=AB and DQ= AC. Join PQ.

Proof: In Δ ABC and Δ DPQ



AB=DP	By Construction
AC=DQ	By Construction
∠ A =∠D	Given
Δ ABC ≅ Δ DPQ→Eq3	By SAS Congruency Rule

From Eq1, $\frac{AB}{DE} = \frac{AC}{DF} \Rightarrow \frac{DP}{DE} = \frac{DQ}{DF}$ (∵DP=AB and DQ=AC)

⇒ PQ∥EF (By Converse of Basic Proportionality Theorem)

∠DPQ =∠E and ∠DQP =∠F → Eq 2

(∵ Corresponding angles are equal)

∴ ΔDPQ~ Δ DEF(By AA Similarity criterion)→Eq4

From Eq 3and Eq4 we get,

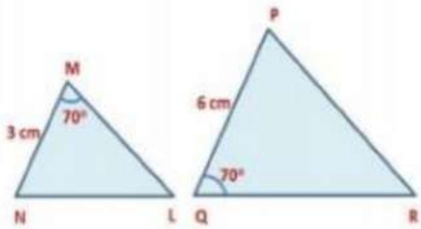
ΔABC~ Δ DEF

(Since two congruent triangles are always similar but its converse is not true)

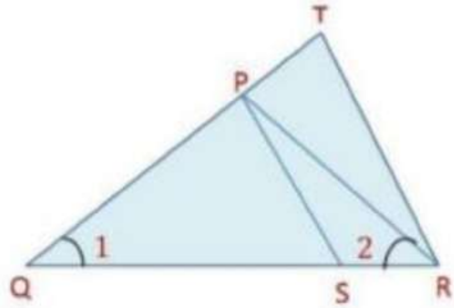
Example: For the given pair of triangles, state the similarity criterion used and write the relation in symbolic form.

In ΔMNL and ΔQPR

$\angle M = \angle Q = 70^\circ$
$\frac{MN}{PQ} = \frac{1}{2}$
$\frac{ML}{QR} = \frac{1}{2}$
Then, $\frac{MN}{PQ} = \frac{ML}{QR}$
$\therefore \Delta$ MNL$\sim\Delta$ QPR by SAS similarity Criterion



Example: In the given figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$. Show that $\Delta PQS \sim \Delta TQR$.



In ΔPQR, $\angle 1 = \angle 2$ (given

PR = PQ

(Sides opposite to equal angles of a triangle are also equal)

$\frac{QR}{QS} = \frac{QT}{PR}$ (Given)

Now, PR = PQ$\Rightarrow$ $\frac{QR}{QS} = \frac{QT}{PQ}$ $\rightarrow$Eq 1

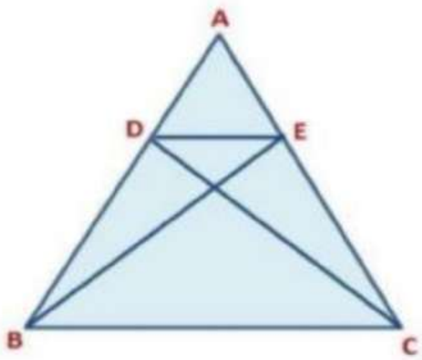
Taking reciprocal of Eq 1 we get, $\frac{QR}{QS} = \frac{PQ}{QT}$$\rightarrow$Eq 2

In ΔPQS and ΔTQR,

∠PQS = ∠TQR	Common
$\frac{QR}{QS} = \frac{QT}{PQ}$	FromEq 2
ΔPQS ~ ΔTQR	By SAS similarity criterion

Example: In the given figure, if Δ ABE≅ Δ ACD, show that ΔADE~ ΔABC.

(REFERENCE: NCERT)



It is given that Δ ABE≅ Δ ACD

⟹ AB = AC and AE = AD

(Corresponding parts of congruent triangles)

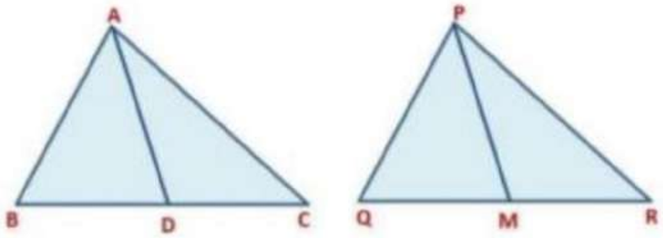
$\frac{AB}{AC} = 1$ and $\frac{AD}{AE} = 1 \Rightarrow \frac{AB}{AC} = \frac{AD}{AE}$→Eq 1

In ΔADE and ΔABC, we have $\frac{AD}{AE} = \frac{AB}{AC}$ (From Eq 1)

$\Rightarrow \frac{AD}{AB} = \frac{AE}{AC}$ and ∠DAE = ∠BAC (common angle)

ΔADE ~ ΔABC by SAS similarity criterion

Example: Sides AB and BC and median AD of Δ ABC are respectively proportional to sides PQ and QR and median PM of ΔPQR. Show that ΔABC ~ ΔPQR.



It is given that BC and AD are medians

ofΔ ABC and Δ PQR respectively,

and $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$

Now, $\frac{AB}{PQ} = \frac{1/2BC}{1/2QR} = \frac{AD}{PM}$ (multiplying both numerator and denominator of the middle term by 1/2)

$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

(Median bisects the opposite side, 1/2 BC = BD and 1/2 QR = QM)

∴ ΔADB~ ΔPMQ (by SSS Similarity Criterion)

⇒ ∠B = ∠Q →Eq 1

Area of similar triangles

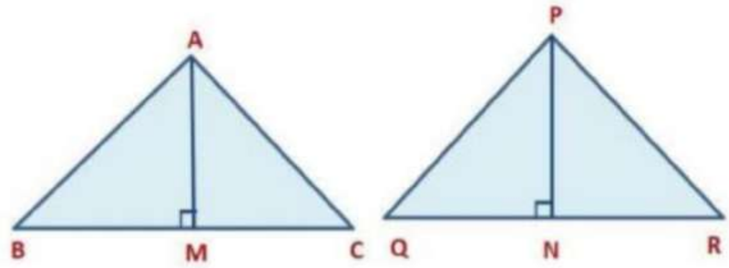
Areas of Similar Triangles

The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Given: ΔABC~ ΔPQR

To prove : $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = (\frac{AB}{PQ})^2 = (\frac{BC}{QR})^2$

CONSTRUCTION : Draw altitudes AM and PN of ΔABC and ΔPQR.



Proof:

$$ar(\triangle ABC) = \frac{1}{2} X BC X AM$$

$$ar(\triangle PQR) = \frac{1}{2} X QR X PN$$

$$\frac{ar(\triangle ABC)}{ar(\triangle PQR)} = \frac{\frac{1}{2} X BC X AM}{\frac{1}{2} X QR X PN} = \frac{BC X AM}{QR X PN} \rightarrow Eq1$$

In ΔABM and ΔPQN

∠B = ∠Q	As ΔABC ~ ΔPQR
∠M = ∠N	Each is of 90°
ΔABM~ ΔPQN	By AA similarity criterion

$\therefore \frac{AM}{PN} = \frac{AB}{PQ}$ →Eq 2 also ΔABC~ ΔPQR (given)

$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$ →Eq 3

$$\frac{ar(\triangle ABC)}{ar(\triangle PQR)} = \frac{AB}{PQ} X \frac{AM}{PN} \quad (\because \frac{BC}{QR} = \frac{AB}{PQ})$$

$= \frac{AB}{PQ} X \frac{AB}{PQ}$ (From Eq 2)

$= = (\frac{AB}{PQ})^2$

Using Eq 3, we get:

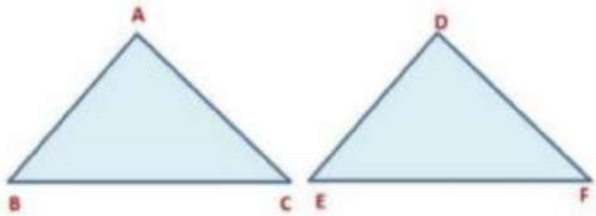
$$\frac{ar(\triangle ABC)}{ar(\triangle PQR)} = (\frac{AB}{PQ})^2 = (\frac{AC}{PR})^2 = \left(\frac{BC}{QR}\right)^2$$

Example: Let ΔABC ~ ΔDEF and their areas be respectively 64 cm2 and 121 cm2.

If EF = 15.4 cm, then find BC.

(REFERENCE: NCERT)

Given: ΔABC ~ ΔDEF



$$\therefore \frac{ar(\triangle ABC)}{ar(\triangle PQR)} = (\frac{BC}{EF})^2$$

(By a theorem of the area of similar triangles)

$$\frac{64}{121} = (\frac{BC}{15.4})^2 \Rightarrow (\frac{8}{11})^2 = (\frac{BC}{15.4})^2 \Rightarrow \frac{8}{11} = \frac{BC}{15.4} \Rightarrow BC$$

$$= \frac{8X15.4}{11}$$

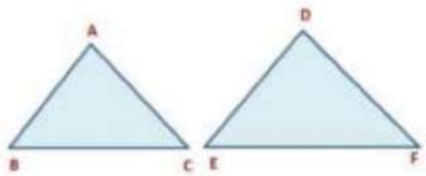
BC = 11.2 cm

Example: If the areas of two similar triangles are equal, then prove that they are congruent.

Let the two triangles be ABC and DEF

It is given that ΔABC~ ΔDEF and

ar(ΔABC) = ar(ΔDEF)



Therefore, $\frac{ar(\triangle ABC)}{ar(\triangle PQR)} = 1$

$(\frac{AB}{DE})^2 = (\frac{AC}{DF})^2 = (\frac{BC}{EF})^2$

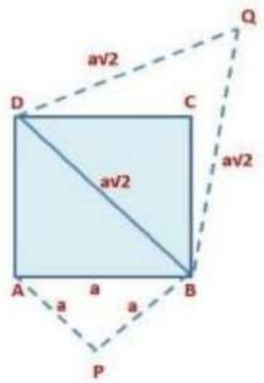
(By a theorem of the area of similar triangles)

⇒ AB = DE, BC = EF and CA = FD

∴ ΔABC≅ ΔDEF (By SSS Congruency)

Example: Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of an equilateral triangle described on one of its diagonals.

(REFERENCE: NCERT)



Let ABCD be a square of length a.

We know, $BD^2 = AB^2 + AD^2 \Rightarrow BD^2 = a^2 + a^2 = 2a^2$

BD = $\sqrt{2a}$

Now, construct two equilateral triangles described on one side of the square and another described on the diagonal of the square.

∴ ΔPAB~ ΔQBD (Equilateral triangles are similar)

$$\therefore \frac{ar\,(\Delta PAB)}{ar\,(QBD)} = \left(\frac{AB}{BD}\right)^2 = \frac{a^2}{\left(a\sqrt{2^2}\right)} = \frac{a^2}{2a^2} = \frac{1}{2}$$

(By a theorem of the area of similar triangles)

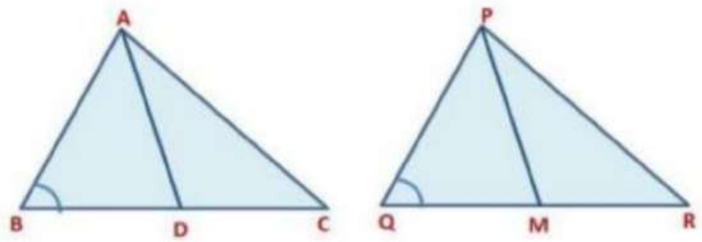
$ar(\Delta PAB) = \frac{1}{2}\, ar(\Delta QBD)$

Example: Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

(REFERENCE: NCERT)

Let ΔABC and ΔPQR be the two similar triangles.

Let AD and PM be the median of ΔABC



And ΔPQR respectively.

We know that ΔABC ~ ΔPQR

⇒ ∠B = ∠Q →Eq 1 (Corresponding angles are equal)

$\frac{AB}{PQ} = \frac{BC}{QR}$ (Ratio of corresponding sides are equal)

$$\Rightarrow \frac{AB}{PQ} = \frac{2BD}{2QM}$$

(As D is the mid-point of BC and M is the mid-point of QR)

$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM}$ →Eq 2

In ΔABD and ΔPQM,

∠ABD = ∠PQM	From Eq 1
$\frac{AB}{PQ} = \frac{AD}{PM}$ →Eq 3	The ratio of corresponding sides are equal
ΔABD~ ΔPQM	By SASsimilarity criterion

Now, $\frac{ar(\triangle PAB)}{ar(\triangle QBD)} = (\frac{AB}{BD})^2$ (By a theorem of the area of similar triangles)

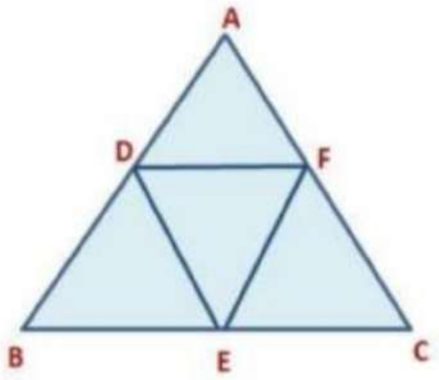
$\therefore \frac{ar(\triangle PAB)}{ar(\triangle QBD)} = (\frac{AD}{PM})^2$ = (ADPM)2 (From Eq 3)

Example: D, E, and F are respectively the mid-points of sides AB, BC and CA of ΔABC. Find the ratio of the areas of ΔDEF and ΔABC.

(REFERENCE: NCERT)

It is given that D, E, and F are the mid-points of sides AB, BC, and CA. Join the points D, E, and F.

We know that the line segment joining the mid-points of two sides of a triangle is parallel to the third side and its length is equal to the half of the length of the third side (Mid-Point Theorem)



$DF = \frac{1}{2}BC, DE = \frac{1}{2}CA, and EF = \frac{1}{2}AB$ →Eq 1

In ΔDEF and ΔCAB,

$\frac{DF}{BC} = \frac{DE}{CA} = \frac{EF}{AB} = \frac{1}{2}$ (From Eq 1)

ΔDEF ~ ΔCAB (by SSS Similarity Criterion)

Now, $\frac{ar(\triangle DEF)}{ar(\triangle CAB)} = (\frac{DE}{CA})^2$ (By a theorem of the area of similar triangles)

$= \frac{(\frac{1}{2}CA)^2}{CA} = \frac{1}{4}$

$\therefore \frac{ar(\triangle DEF)}{ar(\triangle ABC)} = \frac{1}{4}$

Pythagoras Theorem

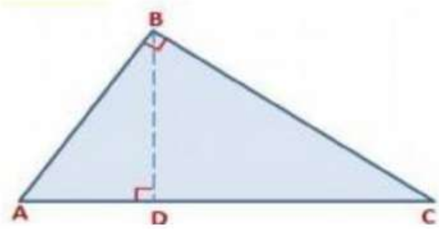
Pythagoras Theorem is also known as the Baudhayan Theorem. Now we will prove this theorem using the concept of similarity of two triangles formed by the perpendicular to the hypotenuse from the opposite vertex of the right-angled triangle.

Theorem 1: If a perpendicular is drawn from the vertex of the right angle of a right-angled triangle to the hypotenuse, then triangles on both sides of the perpendicular are similar to the whole triangle and each other.

Given: A right triangle ABC, right-angled at B. Let BD be the perpendicular to the hypotenuse AC.

To Prove:

i) ΔADB~ ΔABC
ii) ΔBDC~ ΔABC
iii) ΔADB~ ΔBDC
Proof: In ΔADB and ΔABC

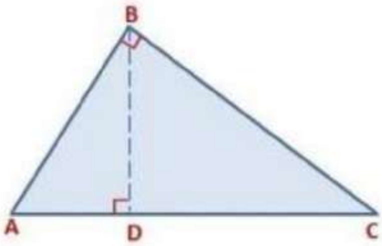


∠A =∠A	Common Angle
∠ADB = ∠ABC	90^0
ΔADB~ ΔABC	By AA similarity criterion

Similarly, ΔBDC ~ ΔABC, ΔADB~ ΔBDC
We will apply this theorem in proving the Pythagoras Theorem:

Pythagoras Theorem
In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Given: A right triangle ABC right angled at B
To Prove: $AC^2 = AB^2 + BC^2$
Construction: Draw BD⊥ AC
Proof: As ABC is a right-angled triangle and BD⊥ AC.
∴ ΔADB ~ ΔABC (By theorem 1)



So, $\frac{AD}{AB} = \frac{AB}{AC}$

(Corresponding sides of similar triangles are proportional)

⟹ AD. AC = AB^2 →Eq 1

Also, ΔBDC ~ ΔABC (By theorem 1)

So, $\frac{CD}{BC} = \frac{BC}{AC}$

(Corresponding sides of similar triangles are proportional)

$\Rightarrow CD.\ AC = BC^2 \rightarrow$ Eq 2

Adding Eq 1 and Eq 2 we get,

$AD.\ AC + CD.\ AC = AB^2 + BC^2 \Rightarrow AC(AD + CD) = AB^2 + BC^2$

$AC.\ AC = AB2 + BC^2$

$(\because AC = AD + CD)$

$AB^2 + BC^2 = AC^2$

The above theorem was given by an ancient Indian Mathematician Baudhayan in the following form:

The diagonal of a rectangle produces by itself the same area as produced by its both sides (i.e. length and breadth). This theorem is referred to as the Baudhayan Theorem.

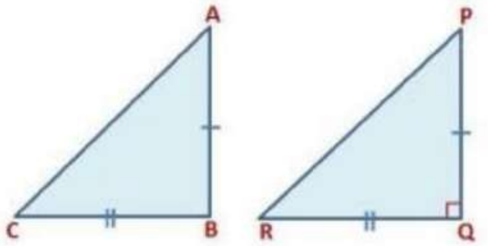
The converse of Pythagoras Theorem
In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

Given: A triangle ABC in which $AC2 = AB2 + BC2$
To Prove: $\angle B = 90°$
Construction: Construct a Δ PQR right-angled at Q, such that PQ = AB and QR = BC
Proof: From Δ PQR we have,

$$PR^2 = PQ^2 + QR^2$$

(Using Pythagoras theorem as $\angle Q = 90°$)

Now, $PR^2 = AB^2 + BC^2 \rightarrow$ Eq 1
(By Construction PQ = AB and QR = BC)
and$AC2 = AB^2 + BC^2 \rightarrow$ Eq 2 (Given)



$\therefore$ AC = PR (From Eq 1 and Eq 2)
In $\Delta ABC \sim \Delta PQR$

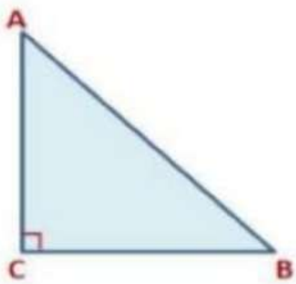
AB = PQ	By Construction
BC=QR	By Construction
AC = PR	Proved above
$\Delta ABC \cong \Delta PQR$	By SSScongruence rule

$\therefore \angle B = \angle Q$ (By CPCT)

But $\angle Q = 90°$(By Construction)

So, $\angle B = 90°$

Example: In an isosceles right-angled triangle, if the hypotenuse is $7\sqrt{2}$ cm, then find the length of the sides of the triangle.
Let ABC be an isosceles right-angled triangle, right-angled at B with AB = AC.



Now, AC = $7\sqrt{2}$ cm

By Pythagoras Theorem,

$AC^2 = AB2 + BC^2$

$AC^2 = AB^2 + AB^2$ (Given AB = BC)

$AC^2 = 2AB^2 \Rightarrow (7\sqrt{2})\ 2 = 2AB^2 \Rightarrow 98 = 2AB^2 \Rightarrow 49 = AB^2$

$AB = 7$ cm
$\therefore AB = BC = 7$ cm

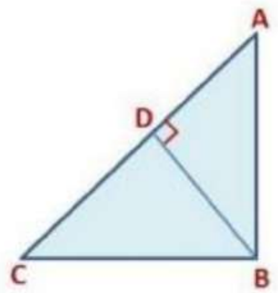
Example: In the given figure, if $AD \perp BC$, prove that

$$AB^2 + CD^2 = BD^2 + AC^2$$

In right-angled ΔBDA, $AB^2 = AD^2 + BD^2 \rightarrow$Eq 1
(By Pythagoras Theorem)

In right-angled ΔCDA, $AC^2 = CD^2 + AD^2 \rightarrow$Eq 2
(By Pythagoras Theorem)

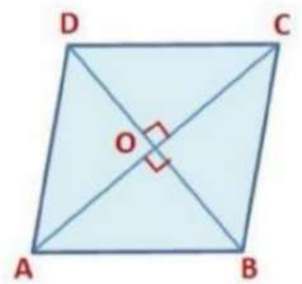
Subtracting Eq 2 from Eq 1 we get,
$AB^2 - AC^2 = AD^2 + BD^2 - (CD^2 + AD^2)$
$AB^2 - AC^2 = AD^2 + BD^2 - CD^2 - AD^2$
$\Rightarrow AB^2 - AC^2 = BD^2 - CD^2$
$AB^2 + CD^2 = BD^2 + AC^2$



Example: Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

(REFERENCE: NCERT)

Let ABCD be rhombus in which AB = BC = CD = DA Diagonals AC and BD are right angle bisectors of each other at O.
In Δ AOB, $\angle AOB = 90°$



OA= $\frac{1}{2}$ AC ($\because$ OA = OC) →Eq 1

And OB= $\frac{1}{2}$ BD ($\because$ OB = OD) →Eq 2

Using Pythagoras theorem, we get

$$OA^2 + OB^2 = AB^2$$

From Eq 1 and Eq 2

$(\frac{1}{2}AC)^2 + (\frac{1}{2}BD)^2 = AB^2 \Rightarrow AC^2 + BD^2 = 4AB^2$

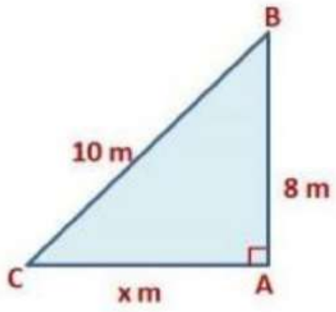
$\Rightarrow AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

($\because$ AB = BC = CD = DA)

Example: A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from the base of the wall.

(REFERENCE: NCERT)

Let AB be the position of a window from the ground, then the height of the window, AB = 8 m and length of the ladder BC = 10 m
Let AC = x m be the distance of the foot of the ladder from the base of the wall.
In Δ BAC, using Pythagoras Theorem we get,



$BC^2 = AC^2 + AB2$

$10^2 = x^2 + 8^2 \Rightarrow 100 = x^2 + 64 \Rightarrow 36 = x^2$

$x = 6$
$AC = 6$ m

∴ The distance of the foot of the ladder from the base of the wall is 6 m.

The converse of Pythagoras Theorem

In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

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To Prove: $\angle B = 90°$

Construction: Construct a Δ PQR right-angled at Q, such that PQ = AB and QR = BC

Proof: From Δ PQR we have,